

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

EME4086 - FINITE ELEMENT METHOD
(ME)

15 OCTOBER 2016
2.30 p.m.- 4.30 p.m.
(2 Hours, Open Book)

INSTRUCTION TO STUDENTS

1. This question paper consists of 6 pages with 5 Questions.
2. Questions 3 to 4 are **OPTIONAL**. Attempt only **ONE** out of the **TWO** questions.
3. Questions 1, 2 and 5 are **COMPULSORY**. You **MUST** attempt these questions.
4. All questions carry equal marks and the distribution of the marks for each question is given.
5. Please write all your answers in the Answer Booklet provided.

COMPULSORY
Question 1:

Figure Q1 shows the cross-section of a composite bar. Two concentrated loads, each of $P = 20\text{kN}$, is applied to the centre rigid plate as shown. The composite bar is made of an aluminium rod in a steel tube which joins to a steel rod. They are all circular with cross-sectional areas below:

Area of the Aluminium Rod = 1200 mm^2

Area of the Steel Tube = 700 mm^2

Area of the Steel Rod = 1600 mm^2

The Young's Modulus of steel is 200 GPa and that of aluminium is 70 GPa . The bar is pinned at the left wall and fixed on the right.

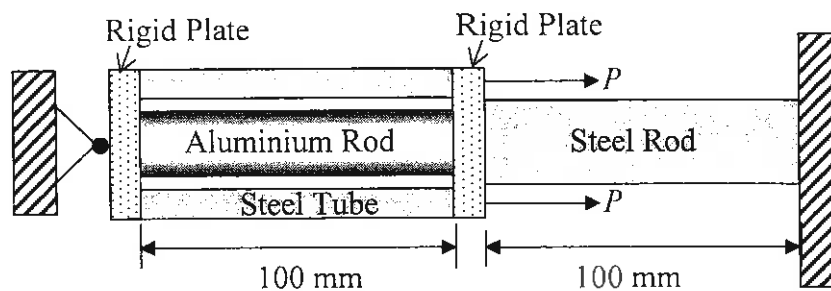


Figure Q1

- a) Using the minimum number of finite elements, propose a ONE-DIMENSIONAL linear finite-element model for this problem by representing the composite bar and steel rod with one-dimensional elements. Please draw the model in your answer script with nodes and elements numbered clearly. Specify the boundary conditions in your sketch as well. **[5 marks]**
- b) For the proposal in part a):
 - i) Calculate the stiffness matrix for each element. **[6 marks]**
 - ii) Write down the finite element equation in the form of $[K][U] = [F]$. The matrix $[K]$, and the vectors $[F]$ and $[U]$ should be expressed in numerical values where applicable. **[9 marks]**
 - iii) Solve for unknowns U and F . **[4 marks]**
 - iv) Make a quick analysis using static equilibrium to confirm your calculations. **[1 mark]**

Continued...

COMPULSORY
Question 2:

Consider the governing equation with boundary conditions below:

$$-\frac{d}{dx} \left((1+2x^2) \frac{du}{dx} \right) - u = x^2 \quad \text{for } 0 \leq x \leq 1 \quad (\text{Eq. 2.1})$$

$$u(0) = 1, \quad \left(\frac{du}{dx} \right)_{x=1} = 2$$

- a) Show, in detailed steps, that the weak form with trial function v is

$B(v, u) = l(v)$ where

$$B(v, u) = \int_0^1 - \left[(1+2x^2) \frac{dv}{dx} \frac{du}{dx} + vu \right] dx \quad \text{and}$$

$$l(v) = \int_0^1 vx^2 dx - 6v(1)$$

[7 marks]

- b) Propose an appropriate function ϕ_0 and a trigonometric trial function ϕ_1 for use in the Galerkin's method such that $u = \phi_0 + c_1 \phi_1$. Explain how ϕ_0 and ϕ_1 satisfy the boundary conditions. DO NOT SOLVE for c_1 . [6 marks]

- c) Using the Ritz's method, the same ϕ_0 in part b) and assume $\phi_1 = x$.

i) Explain how ϕ_1 meets the necessary requirements. [2 marks]

ii) Given $u = \phi_0 + c_1 \phi_1$. Solve for c_1 and rewrite u in terms of variable x .

[10 marks]

Continued...

OPTIONAL
Question 3:

The governing equation and boundary conditions for a fin which is experiencing conduction and convection heat transfer are given as:

$$-\frac{d^2\theta}{dx^2} + c\theta = 0, \quad 0 < x < L \quad (\text{Eq. 3.1})$$

$$\theta(0) = T_0 - T_\infty, \quad \left[k \frac{d\theta}{dx} + \beta\theta \right]_{x=L} = 0$$

Use the following data:

$$T_0 = 100^\circ\text{C}, \quad T_\infty = 25^\circ\text{C}, \quad L = 0.2\text{m}, \\ c = 300/\text{m}^2, \quad k = 50\text{W}/(\text{m}\cdot\text{K}) \quad \text{and} \quad \beta = 60\text{W}/(\text{m}^2\cdot\text{K})$$

- a) Propose a ONE-DIMENSIONAL linear finite-element model on the domain by:
 - i) Identifying natural and essential boundary conditions of this problem; [2 marks]
 - ii) Representing the bar with two equal-interval one-dimensional elements. Please draw the model in your answer script with nodes and elements numbered clearly. Specify the boundary conditions in your sketch as well. [3 marks]
- b) Calculate the temperatures at $x = L/2$ and $x = L$ by constructing $[K][U]=[F]$. [14 marks]
- c) Estimate the temperature at $x = L/4$, which does not coincide with the node location. What is your main assumption for the estimation, and why is it appropriate? [4 marks]
- d) Based on governing equation of this problem and your analysis in part c), explain your rationale if increasing the number of linear finite elements in part a) ii) will alter the solution in part b). [2 marks]

Continued...

OPTIONAL

Question 4:

A truss shown in **Figure Q4** is supported by a spring at node 1. Each member is made of the same material (Young's Modulus, $E = 200$ GPa, Yield Stress = 30 MPa), and with constant cross-sectional area, $A = 5.0 \times 10^4 \text{ m}^2$. The elements and nodes are numbered, and all other dimensions are shown in the **Figure Q4**.

Using the minimum number of elements,

- Write the assembled stiffness matrix $[K]$, the displacement vector $[U]$ and force vector $[F]$. **[12 marks]**
- Solve for any unknown displacements in vector $[U]$. **[7 marks]**
- Analyse if any of the truss members has yielded. **[6 marks]**

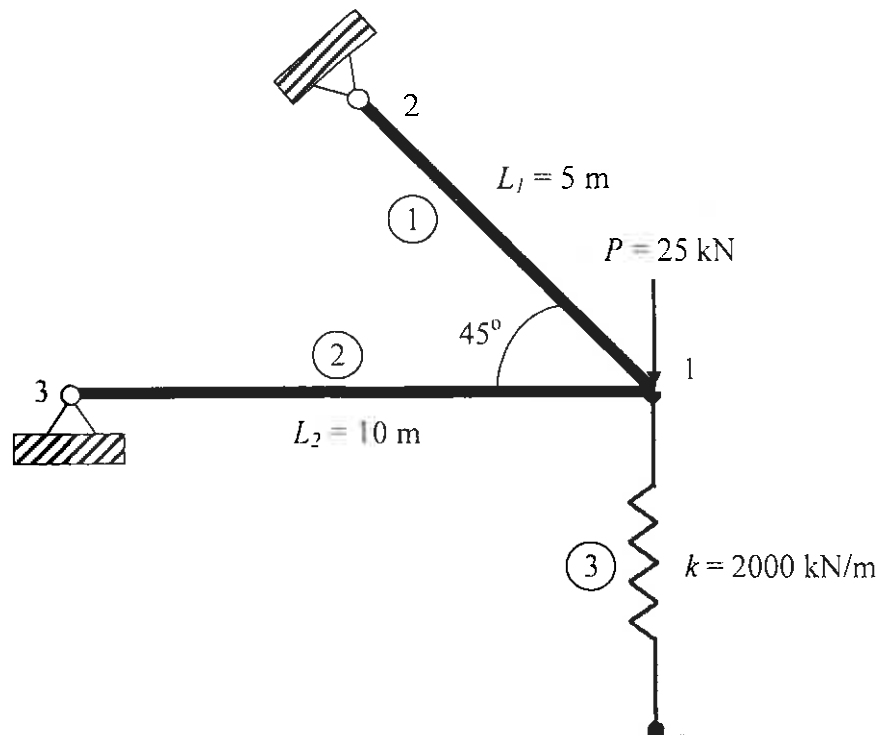


Figure Q4

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COMPULSORY
Question 5:

A two-dimensional square plate with dimensions of 0.2 m and isotropic thermal conductivity, $k = 100 \text{ W/(m}^\circ\text{C)}$ is shown in **Figure Q5**. The square plate is subjected to a uniform temperature of 100°C at three sides, namely side 1-2, side 3-4, and side 4-1. Side 2-3 is exposed to ambient with convective heat transfer as shown in the **Figure Q5**.

The square plate has been discretized into 4 identical triangles as shown. Elements and global nodes are all labelled, within and without circles, respectively. The heat transferred phenomena can be represented by finite element equation of the form $[K][U] = [F]$.

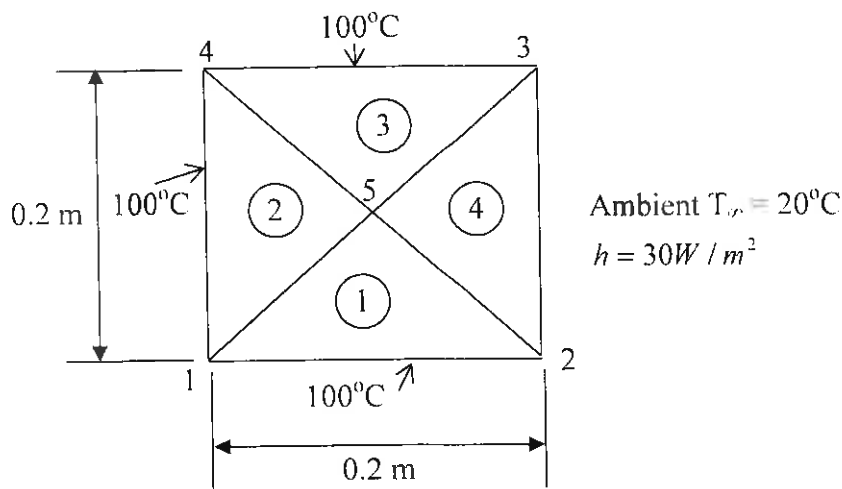


Figure Q5

- a) Accounting for the convective heat transfer along side 2-3, determine the stiffness matrix $[K]$ of element (1) with node connectivity 1-2-5. [13 marks]
- b) Write the assembled stiffness matrix $[K]$ for the entire domain. [8 marks]
- c) Express the corresponding vectors $[U]$ and $[F]$ in numerical values, where applicable. [4 marks]

End of Paper.